Numerical solution of high-dimensional partial differential equations using Bernoulli Wavelets

SHORT SYNOPSIS

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ABSTRACT

This work focuses on solving high-dimensional partial differential equations with various initial and boundary conditions using Bernoulli wavelets within a wavelet-based numerical framework, supported by Kronecker product and vectorization techniques. The reliability and computational performance of the proposed method are assessed through CPU time, absolute error estimation, and convergence analysis. The synopsis is divided into seven sections.

Section 1 Presents a concise overview of the historical development of wavelet theory and is organized into four subsections. **Subsection 1.1** Provides the basic definitions of wavelets together with examples, offering important foundational knowledge for the reader. **Subsection 1.2** Presents multi-resolution analysis (MRA), the development of wavelets from MRA. **Subsection 1.3** Provides an overview of wavelet-based numerical methods applied to differential equations. And **subsection 1.4** Describes the definition along with the fundamental properties of the Kronecker product.

Section 2, *Literature Review,* presents a summary of the significant contributions of various authors related to the present study.

Section 3 provides Rationale/ Significance of the Study.

Section 4 is Problem Statement.

Section 5 outlines the objectives and goals of the research.

Section 6 focuses on the analysis of the research methodology.

Sections 7 and 8 discuss the expected outcomes of the research and present the list of references, respectively.

Keywords: Bernoulli Wavelets, Wavelet-based Numerical Method, Kronecker product, Vectorization Operator, Numerical Solution, Partial differential equations.

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1 Introduction

Partial differential equations (PDEs) have a significant role in modeling, a wide range of physical engineering and biological phenomena including heat conduction, fluid flow, wave propagation, population dynamics and financial systems. The solution of complex high-dimensional PDEs relies on effective numerical methods, while recent progress in engineering and applied mathematics continues to improve computational problem-solving frameworks.

Wavelet-based numerical methods have gained increasing attention in recent decades due to their strong approximation properties and computational efficiency. Wavelets differ from classical polynomial-based methods by providing localized basis functions that efficiently represent both global patterns and local details of solutions. Consequently, wavelet methods are highly effective for PDEs exhibiting sharp gradients, boundary layers, and multiscale behavior. Wavelet-based numerical methods form a framework in which functions are represented through basis functions known as wavelets. These wavelets possess localization in both space and time, and the method commonly makes use of compactly supported wavelets [1].

The proposed approach employs Bernoulli wavelets generated from classical Bernoulli polynomials, which were introduced by Jakob Bernoulli in *Ars Conjectandi (1713)* during his study of sums of integer powers. In modern numerical analysis, these polynomials are transformed into a wavelet basis by confining them to the interval [0,1] and applying suitable dilation and translation, as presented by Keshavarz et al. [2]. Wavelets are constructed using the Multiresolution Analysis (MRA) technique, which was introduced by Ingrid Daubechies in 1992 [3].

Bernoulli wavelets with Galerkin and collocation techniques has proven to be an efficient approach for solving linear and nonlinear PDEs. The unknown solution can be expanded using Bernoulli wavelet bases, and with the use of suitable projection techniques, high accuracy is achieved using a relatively small number of basis functions. Bernoulli wavelet methods are also flexible and can be applied to high-dimensional problems, variable coefficients, fractional derivatives, and time-dependent boundary conditions.

These advantages, encourages to focuses on the numerical solution of high dimensional partial differential equations using Bernoulli wavelets. The proposed approach offers an efficient and accurate framework for solving various high-dimensional PDEs with reduced computational complexity. Numerical examples are provided to verify the effectiveness and convergence properties of the proposed Bernoulli wavelet-based method. The Kronecker product [4] and vectorization operator are utilized within the Bernoulli wavelet framework to enable the method to handle high-dimensional partial differential equations.

1.1 Wavelets

The concept of wavelets was introduced in 1980 by Grossmann and Morlet [5]. Fourier analysis relies on oscillatory basis functions that are not effective for signals with sudden changes. A wavelet is a function that, along with its dilated and translated forms, can be used to represent a wide class of functions. The Haar wavelet, introduced in 1910, represents the simplest wavelet. In the early 1980s, wavelets capable of adapt-

ing to dif-ferent components of a signal were investigated to better approximate data with sharp discontinuities. In 1989, Stéphane G. Mallat developed the concept of multiresolution analysis (MRA) for constructing orthonormal wavelets. In the early 1980s, Jean Morlet de-veloped the wavelet transform as a significant mathematical tool for seismic data analy-sis. It was designed to overcome the limitations of Fourier methods in handling non-stationary signals by providing simultaneous time–frequency localization using scaled and translated forms of a mother wavelet [6].

By $L^1(\mathbb{R})$, the set of all measurable complex valued functions f defined on $\mathbb{R}$ for which

$$\int_{\mathbb{R}} |f(x)|\, dx < \infty,$$

and by $L^2(\mathbb{R})$, the set of all measurable complex valued functions f defined on $\mathbb{R}$ for which

$$\int_{\mathbb{R}} |f(x)|^2\, dx < \infty.$$

Almost identical functions are identified. Using usual addition and scalar multiplication of functions along with the inner product $\langle f, g\rangle$ of $f, g \in L^2(\mathbb{R})$ defined by

$$\langle f, g\rangle = \int_{\mathbb{R}} f(x)\overline{g(x)}\, dx,$$

$L^2(\mathbb{R})$ forms a Hilbert space.

Definition 1.1.1. [6] The Fourier Transform of a function $f : L^2(\mathbb{R}) \to L^2(\mathbb{R})$ is defined as

$$\hat{f}(\xi) = \int_{\mathbb{R}} f(x)e^{-i\xi x}dx. \;\; x,\ \xi \in \mathbb{R} \tag{1.1}$$

Definition 1.1.2. [6] The inverse Fourier Transform of a function $g(x) : L^2(\mathbb{R}) \to L^2(\mathbb{R})$ is defined as

$$\check{g}(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} g(\xi)e^{ix\xi}d\xi, \quad x,\ \xi \in \mathbb{R} \tag{1.2}$$

If $g = \hat{f}$, then $(\check{\hat{f}}) = f$

Definition 1.1.3. [6] (**Wavelets on** $L^2(\mathbb{R})$) An orthonormal wavelet on $L^2(\mathbb{R})$ is a function $\psi \in L^2(\mathbb{R})$ such that $\psi_{j,k}$ is an orthonormal basis of $L^2(\mathbb{R})$, where

$$\psi_{j,k}(x) = 2^{\frac{j}{2}}\psi(2^j x - k),\ j, k \in \mathbb{Z}. \tag{1.3}$$

This system is generated from ψ, by translating it by the integers k, applying the dyadic dilations 2^j, to these translates.

Definition 1.1.4. (Mother Wavelets)

A mother wavelet is a function $\psi(x)$ that satisfies the following conditions:

1. $C_\psi = \int_0^\infty \frac{|\psi(w)|^2}{|w|}dw$ where $\psi(w)$ is the Fourier transform of $\psi(x)$ as a result

$$\int_{-\infty}^{\infty} \psi(x)dx = 0$$

2. A wavelet function is of unit energy

$$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$$

A wavelet $\psi_{a,b}(x)$ is constructed from a mother wavelet by scaling and translation.

$$\psi_{a,b}(x) = \frac{1}{\sqrt{|a|}}\psi\left(\frac{x-b}{a}\right), \quad a, b \in \mathbb{R}, a \neq 0$$

Here, a controls the scale (compression), and b sets the wavelet's position along the x-axis.

Examples of Wavelets:

1. Haar Wavelet: The Haar basis of $L^2(\mathbb{R})$ discovered by Haar in 1910 [8] describes an orthonormal wavelet. For the function ψ defined by

$$\psi_H(x) = \begin{cases} 1 & \text{if } 0 \leq x < \frac{1}{2} \\ -1 & \text{if } \frac{1}{2} \leq x < 1 \\ 0 & \text{otherwise,} \end{cases}$$

the system $\{\psi_H(x)j, k : j, k \in \mathbb{Z}\}$ forms an orthonormal basis for $L^2(\mathbb{R})$.

2. Shannon Wavelet: The function $\psi_S \in L^2(\mathbb{R})$ which satisfies $\hat{\psi}_S(\xi) = \chi_{I_S}(\xi)$, where

$$I_S \equiv \left[-1, -\frac{1}{2}\right) \cup \left[\frac{1}{2}, 1\right),$$

is called the *Shannon Wavelet.*

3. Journe Wavelet: The function $\psi_J \in L^2(\mathbb{R})$ which satisfies $\hat{\psi}_J(\xi) = \chi_{W_J}(\xi)$, where

$$W_J \equiv \left[-\frac{16}{7}, -2\right) \cup \left[-\frac{1}{2}, -\frac{-2}{7}\right) \cup \left[\frac{2}{7}, \frac{1}{2}\right) \cup \left[2, \frac{16}{7}\right),$$

is called the *Journe Wavelet.*

4. Meyer Wavelet: If a function $\Theta(\xi)$ defined on $\mathbb{R}$ satisfies the following five properties:

(a) $0 \leq \Theta(\xi) \leq 1$,

(b) $\Theta(\xi) = \Theta(-\xi)$,

(c) $\Theta(\xi) = 1$ for $|\xi| < \frac{1}{3}$,

(d) $\Theta(\xi) = 0$ for $|\xi| > \frac{2}{3}$,

(e) $\Theta^2(\xi) + \Theta^2(\xi - 1) = 1$ for $0 \leq \xi \leq 1$,

then the 1-periodic function $m(\xi) = \Theta(2\xi)$ for $\xi \in \left[-\frac{1}{2}, \frac{1}{2}\right)$ together with Θ gives rise to a wavelet ψ given by

$$\hat{\psi}(\xi) = e^{\pi i \xi}\, m\left(\frac{\xi}{2} + \frac{1}{2}\right)\, \Theta\left(\frac{\xi}{2}\right)$$

which satisfies the following properties:

(*i*) supp $\hat{\psi} \subset [-\frac{4}{3}, -\frac{1}{3}] \cup [\frac{1}{3}, \frac{4}{3}]$

(*ii*) ψ is a real valued smooth function,

(*iii*) $\psi(-\frac{1}{2} - x) = \psi(-\frac{1}{2} + x)$, for all $x \in \mathbb{R}$.

These types of wavelets are known as *Meyer wavelets.* Furthermore, when Θ is smooth and satisfies (a)-(e), $\hat{\psi}$ is also smooth.

5. Daubechies wavelets: Ingrid Daubechies first developed a general method for constructing orthonormal wavelets with compact support and a controllable degree of smoothness. She noted that when ψ and φ lie in $C^{N-1}(\mathbb{R})$, the low-pass filter m_0 can be expressed as

$$m_0^N(\xi) = \left(\frac{1 + e^{-2\pi i\xi}}{2}\right)^N g_N(\xi),$$

with g_N being 1-periodic function and $g_N \in C^{N-1}(\mathbb{R})$.

if $N = 1$, then the periodic function $g_1(\xi) = 1$ and so

$$m_0^1(\xi) = \left(\frac{1 + e^{-2\pi i\xi}}{2}\right)^1$$

.

if $N = 2$, then the periodic function is

$$g_2(\xi) = \frac{1 + \sqrt{3}}{2} + \frac{1 - \sqrt{3}}{2} e^{-2\pi i\xi}$$

and hence the low-pass filter is

$$m_0^2(\xi) = \sum_{k\in\mathbb{Z}} \alpha(k)\, e^{-2\pi i\xi}$$

with

$$\alpha(0) = \frac{1 - \sqrt{3}}{8},\ \alpha(1) = \frac{3 - \sqrt{3}}{8},\ \alpha(2) = \frac{3 + \sqrt{3}}{8},\ \alpha(0) = \frac{1 + \sqrt{3}}{8},$$

and the rest of coefficients are zero.

6. Bernoulli Wavelets: Bernoulli wavelets [2] $\psi_{n,m}(t) = \psi(k, \hat{n}, m, t)$ have four arguments; $\hat{n} = n - 1$, $n = 1, 2, 4, ..., 2^{k-1}$, k can assume any positive integers, m is the order for Bernoulli polynomials and t is the normalized time. We defined them on interval $[0, 1)$ as follows:

$$\psi_{n,m}(t) = \begin{cases} 2^{\frac{k-1}{2}} \beta_m(2^{k-1}t - \hat{n}), & \frac{\hat{n}}{2^{k-1}} \le t < \frac{\hat{n}+1}{2^{k-1}} \\ 0, & otherwise \end{cases}$$

with

$$\beta_m(t) = \begin{cases} 1, & m = 0 \\ \frac{1}{\sqrt{\frac{(-1)^{m-1}(m!)^2}{(2m)!}\alpha_{2m}}} B_m(t), & m > 0 \end{cases}$$

For $m = \{0, 1, 2, 3, ..., M-1\}$ and $n = \{1, 2, 4, ..., 2^{k-1}\}$, the factor $\frac{1}{\sqrt{\frac{(-1)^{m-1}(m!)^2}{(2m)!}\alpha_{2m}}}$ enforces normalization, with the dilation and translation specified by $a = 2^{-(k-1)}$ and $b = \hat{n}2^{-(k-1)}$. Here $B_m(t)$ are the well-known Bernoulli-polynomials of order m [2].

$$B_m(t) = \sum_{i=0}^{m} \binom{m}{i} \alpha_{m-i} t^i$$

where $\alpha_i, i = 0, 1, 2, .., m$ are Bernoulli numbers. These numbers are the sequence of rational numbers which arise in the series expansion of trigonometric functions [15] and can be defined by the identity

$$\frac{t}{e^t - 1} = \sum_{i=0}^{\infty} \alpha_i \frac{t^i}{i!}$$

The first few Bernoulli numbers are

$$\alpha_0 = 1, \alpha_1 = \frac{-1}{2}, \alpha_2 = \frac{-1}{30}, ...$$

with $\alpha_{2i+1} = 0, i = 1, 2, 3, ...$ the first few Bernoulli polynomials are

$$B_0 = 1, B_1 = t - \frac{1}{2}, B_2 = t^2 - t + \frac{1}{6}, B_3 = t^3 - \frac{3}{2}t^2 + \frac{1}{2}t, ...$$

these polynomial satisfies the following formula [15]

$$\int_0^1 B_n(t) B_m(t) dt = (-1)^{n-1} \frac{m!n!}{(m+n)!} \alpha_{n+m}, m, n \geq 1$$

Bernoulli polynomial form a complete basis over the interval [0, 1].

1.2 Multi-resolution Analysis (MRA)[9]

S. Mallat [7] and Y. Meyer [20] introduced the concept of Multi-resolution Analysis. It is a mathematical tool for providing orthonormal wavelets in the space $L^2(\mathbb{R})$.

Definition 1.2.1. [9] A multiresolution analysis is defined by a sequence of closed subspaces $V_j, j \in \mathbb{Z}$ of $L^2(\mathbb{R})$ that satisfy the following properties:

1. $V_j \subset V_{j+1}$ for all $j \in \mathbb{Z}$;
2. $f(x) \in V_j$ if and only if $f(2x) \in V_{j+1}$ for all $j \in \mathbb{Z}$;
3. $\bigcap_{j \in \mathbb{Z}} V_j = \{0\}$;
4. $\overline{\bigcup_{j \in \mathbb{Z}} V_j} = L^2(\mathbb{R})$
5. There exists a function $\phi \in V_0$ such that $\{\phi(x-k) : k \in \mathbb{Z}\}$ is an orthonormal basis for V_0.

The function ϕ is called a scaling function of the given MRA.

1.2.1 Construction of Wavelets from MRA [9]

The goal is to present $L^2(\mathbb{R})$ as a collection of subspaces at various scales. In multiresolution analysis, the subspaces V_j and W_j, represent the scaling function space and the wavelet space, respectively. Let $\{V_j\}$ be a nested sequence of approximation subspaces; the corresponding detail subspace W_j is the orthogonal complement of V_j in V_{j+1}, i.e.,

$$V_j \perp W_j \tag{1.4}$$

$$V_{j+1} = V_j \oplus W_j \tag{1.5}$$

Applying (1.5) recursively, we get

$$\begin{aligned} V_j &= V_{j-2} \oplus W_{j-2} \oplus W_{j-1}, \\ V_J &= V_{J_0} \oplus \bigoplus_{j=J_0}^{J-1} W_J, \ J > J_0. \end{aligned} \tag{1.6}$$

Therefore, any function in V_j can be studied at different scales using (1.6). Continuing the decomposition of (1.6), for $J_0 \to -\infty$ and $J \to \infty$,

$$\bigoplus_{j=-\infty}^{\infty} W_j = L^2(\mathbb{R}) \tag{1.7}$$

The information is passing from V_0 to the space V_1 is represented by translations of $\psi(x)$. For any MRA, there exists a function $\psi_{0,0} = \psi(x) \in W_0$, called the mother wavelet, such that $\{\psi_{j,k}(x) = 2^{\frac{j}{2}}\psi(2^j x - k) : k \in \mathbb{Z}\}$ forms an orthonormal basis for W_j. Since $\psi(x) \in W_0 \subset V_1$,

$$\psi(x) = \sqrt{2} \sum_{k=-\infty}^{\infty} g_k \phi(2x - k). \tag{1.8}$$

Equation(1.8) is known as the wavelet equation, also known as the two-scale relation for the wavelet function. And the coefficients g_k are called the high-pass filter coefficients.

The construction of the mother wavelet function ψ using multi-resolution analysis, requires a sequence of approximation subspaces V_j of $L^2(\mathbb{R})$ (approximation spaces) satisfying definition of MRA and that satisfy the MRA conditions and corresponding closed detail subspaces W_j of $L^2(\mathbb{R})$ satisfying equation (1.7).

The Haar wavelet, the Shannon wavelet, Daubechies and Meyer wavelets are MRA wavelets. Most wavelets are constructed within the framework of a multiresolution analysis. The first example of a wavelet lacking an associated MRA was introduced by J. L. Journé and is now known as the Journé wavelet [7]. This wavelet is band-limited, with its Fourier transform given by a characteristic function. Since this discovery, several non-MRA wavelets have appeared in the literature [10, 11, 14], most of which are band-limited, smooth, but not well localized.

1.3 Wavelets Based Methods for Differential Equations

1.3.1 Traditional Galerkin method [16]:

The origin of the method is generally associated with a paper published by Galerkin (1915) on the elastic equilibrium of rods and thin plates. The fundamental properties

of the Galerkin method are presented in a concise form, assuming a two-dimensional problem defined by a linear differential equation.

$$L(u) = 0 \tag{1.9}$$

in a domain $D(x, y)$, with boundary conditions

$$S(u) = 0 \tag{1.10}$$

on ∂D (the boundary of D). In the Galerkin framework, the solution u is assumed to be accurately approximated by following representation

$$u_a = u_0(x, y) + \sum_{j=1}^{N} a_j \phi_j(x, y) \tag{1.11}$$

Here, the functions $\phi_j's$ are known analytic basis functions, u_0 is introduced to enforce the boundary conditions, and the coefficients $a_j's$ are unknown. Substituting (1.11) into (1.9) produces a nonzero residual R, defined as

$$R(a_1, a_2, ..., a_N, x, y) = L(u_a) = L(u_0) + \sum_{j=1}^{N} a_j L(\phi_j). \tag{1.12}$$

The inner product $< f, g >$ is defined as

$$< f, g >= \int \int_D fg \; dx \; dy \tag{1.13}$$

In the Galerkin method the unknown coefficients, $a_j's$ in (1.12) are obtained by solving the following system of equations:

$$< R, \phi_k >= 0, \quad k = 1, 2, 3, ..., N \tag{1.14}$$

Here R is just the equation residual, and the $\phi_k's$ are the same analytic functions that appear in (1.11). Since this example is based on a linear differential equation, (1.14) can be written directly as a matrix equation for the coefficients a_j as

$$\sum_{j=1}^{N} a_j < L(\phi_j), \phi_k >= - < L(u_0), \phi_k > \tag{1.15}$$

Substitution of the $a_j's$ resulting from the solution of equation (1.15) into equation (1.11) gives the required approximate solution u_a.

1.3.2 Bernoulli-wavelet based Galerkin method [17]:

In Bernoulli-wavelet based Galerkin method (BWGM), trial functions $\phi_j(x, y)'s$ are replaced by Bernoulli-wavelets. The results in BWGM are more accurate.

1.3.3 Collocation Method

The standard collocation approach [12] serves to compute collocation points within a prescribed range $[a, b]$ and is defined as:

$$t_k = a + \frac{(b-a)k}{N}, \quad k = 1, 2, ..., N$$

There are many ways to construct approximation functions according to collocation information, and the choice and definition of these functions will directly affect the stability, accuracy, and applicability of the method. The following content will briefly introduce some commonly used approximation schemes.[13]

- Moving Least Square (MLS) Approximation
- Kernel Approximation
- Polynomial Basis Approximation
- Generalized Finite Difference Approximation

1.4 Kronecker product and vectorization of a matrix [15]

This section outlines the definition and basic properties of the Kronecker product, as described in [15], which will be used later. Consider a matrix A of size $m \times n$ and a matrix B of size $p \times q$, the Kronecker product between two matrices is defined as follows.

$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B & \cdots & a_{1n}B \\ a_{21}B & a_{22}B & \cdots & a_{2n}B \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}B & a_{m2}B & \cdots & a_{mn}B \end{bmatrix}_{mp \times nq}$$

For a $p \times q$ matrix B, its vectorization $vec(B)$ is defined as the $pq \times 1$ column vector obtained by stacking the columns of B in order. When used together, the Kronecker product and vectorization operator exhibit several powerful and convenient matrix identities. Below, we list several important properties of the Kronecker product for the matrices A, C, D and E:

P1. For any scalar λ, $\lambda(C \otimes D) = (\lambda C \otimes D) = (C \otimes \lambda D)$.
P2. (Associativity) $(C \otimes D) \otimes E = C \otimes (D \otimes E)$.
P3. (Distributive over addition) $(C + D) \otimes E = (C \otimes E) + (D \otimes E)$ and $C \otimes (D + E) = (C \otimes D) + (C \otimes E)$.
P4. (Compatibility for matrix multiplication) $(A \otimes C)(D \otimes E) = (AD \otimes CE)$
P5. $(D \otimes E)^{-1} = D^{-1} \otimes E^{-1}$, provided D^{-1} and E^{-1} exist.
P6. $(C \otimes D)^T = C^T \otimes D^T$, where T is the transpose of a matrix.
P7. (Not commutative) $(C \otimes D) \neq (D \otimes E)$
P8. $vec(CAD^T) = (C \otimes D)vec(A)$, where $vec(A)$ is the vectorization operator applied on matrix A.
P9. $vec(C + D) = vec(C) + vec(D)$
P10. If $vec(C) = vec(D)$, then $C = D$.

2 Literature Review

Before 1990, wavelets were used mainly as signal processing tools. The early 1990s marked the first attempt to utilize wavelet bases for numerically solving PDEs, following the development of adaptive wavelet techniques. J. Liandrat and P. Tchamitchian [19] resolved one-dimensional regularized burgers equation using a spatial (spline) wavelet approximation in Galerkin framework.
Y. Maday, et. al., [20] used cubic spline wavelet bases in the Galerkin framework to approximate partial differential equations in 1991.
In 1992, S. Jaffard and P. Laurencot [21] applied wavelet methods for the rapid solution of elliptic problems using the Galerkin ap-proach.
In 1993, S. Qian and J. Weiss [22] used Daubechies wavelet basis in Wavelet-Galerkin to solve PDEs in non-separable do-mains, with a nontrivial adaptation of the standard capacitance matrix method.
In 1994, K. Amaratunga, et. al., [23] used Daubechies wavelet basis in Wavelet-Galerkin solver for the solution of one-dimensional counterpart of Helmholtz's equation (PDE). But, issues related to boundary treatment, operator sparsity, and the efficient solution of time-dependent problems were not fully resolved. And research is limited to one-dimensional problems.
In 1996, M. Q. Chen, et. al., [24] constructed orthonormal Daubechies wavelet to used them in Galerkin method on bounded intervals and calculated connection coefficients on bounded intervals. But at that time Standard Galerkin methods struggle with non-linear terms (like $u\frac{du}{dx}$ in fluid dynamics) because the product of two wavelets is not a single wavelet, but a messy sum of many.
In 1997, J. Frohlich and K. Schneider, [25] extended to solve a nonlinear parabolic PDE using semi-implicit time discretization and self-adaptive wavelet discretization in space. Numerical results with fully adaptive spa-tial wavelet discretization are presented for a one-dimensional flame front and a two-dimensional problem.
In 1999, C. Canuto, et. al. [26], mapped wavelets onto local elements instead of using them on whole domain. This al-lowed the handling of complex geometries (L-shapes, holes) which traditional Wavelet Galerkin methods (defined on rectangles) could not handle.
In 2001, A. Cohen, et. al., [27] proved that an adaptive wavelet solver could achieve a target accuracy of 1ϵ with a computational cost proportional to the number of degrees of freedom N, where N is the absolute minimum needed to represent the solution.
In 2002, K. Urban [28] published his book "Wavelets in Numerical Simulation", to unify the scattered theories. and O. V. Vasilyev and C. Bowman, [29] developed an adaptive numerical method for solving partial differential equations using Second-Generation Wavelet Collocation Scheme.
In 2003, R. S. Stankovic, et. al., [30] described the basic definitions of Haar wavelet transform.
In 2009, E. Deriaz and V. Perrier [31] made WGM highly competitive for $3D$ turbulence simulations by constructed specific vector wavelets that are automatically divergence-free, because the biggest headaches in solving the Navier-Stokes equations is enforcing the condition $\nabla \cdot u = 0$ (divergence-free), which usually requires solving a separate, expensive pressure equation.

In 2010, B. Sarler, et. al., [32] constructed a simple method using Haar wavelets to solve the numerical solution of line-ar and non-linear second-order BVPs arising in the mathematical modeling of different engineering applications. The limitation of the approach in multi-dimensional problems is the increased computational cost due to inversion of $2M \times 2M$ sparse coefficient matrix.
In 2012, E.H. Doha, et. al., [33] derived the shifted Jacobi operational matrix of fractional derivatives and applied it with the spectral tau method to solve general linear multi-term FDEs.
In 2013, F. Haq, et. al., [34] proposed two new methods LWCM and HWCM for numerical solution of EPDEs, and then extended it to parabolic PDEs [35]. And results proved that LWCM is more accurate and LWCM is 8th–9th order convergent whereas HWCM is second order convergent.
In 2014, M. Razzaghi, et. al., [2] solved fractional order differential equations using Bernoulli wavelet operational matrix of fractional order integration.
In 2016, S. Balaji [36] applied the Bernoulli wavelet operational matrix of derivatives in a collocation scheme to solve nonlinear singular Lane–Emden type equations in astrophysics.
In 2017, I. Aziz, et. al., [37] solved three-dimensional elliptic partial differential equations with Dirichlet boundary conditions using collocation method based on Haar wavelet with the help of Kronecker product.
In 2018, Jianrong Zhou, et. al., [38] solved inverse problem of parabolic equation with moving boundaries and integral condition using Ritz–Galerkin method.
In 2019, S.C. Shiralashetti, et. al., [39] used Hermite wavelets in Galerkin Sceme to solve PDEs in one-dimensional space.
In 2021, D. Chouhan, et. al., [40] used Bernoulli wavelet functions and operational matrices of integration to solve Anomalous Infiltration and Diffusion Modeling by Nonlinear Fractional Differential Equations of Variable Order.
In 2022, R. Aruldoss and R. A. Devi [41] developed a novel numerical scheme for Fractional Diffrential Equations With Variable Coefficients, based on the operational matrix of Bernoulli wavelets and also investigate the uniform convergence analysis for Bernoulli wavelets expansion.
In 2023, H. M. Arafa and M. A. Ramadan [42] solved system of Fredholm integral equation of the second kind using Bernoulli wavelet method and provided the Convergence analysis and error estimation of Bernoulli wavelets approach.
In 2024, Mulimani M. and Kumbinarasaiah S. [43] applied Bernoulli wavelet collocation scheme to solve parabolic partial differential equation. And Md. S. Hossan, et. al., [44] solved fractional parabolic partial differential equations using Galerkin-finite difference method.
In 2025, V. Manavi, et. al., [45] Developed the Multiwavelets Galerkin spectral method for fractional partial integro-differential equations. L. Angadi [46] used Bernoulli wavelets as the basis functions in Galerkin method to solve one-dimensional differential equations through the weighted residual technique. And M. Ahmad et. al., [47] included the Kronecker product and vectorization operator properties in the HWM to decreases the computational cost of conventional Haar wavelet methods.

3 Rationale/ Significance of the Study

The study introduces a wavelet-based framework using Bernoulli wavelets that efficiently transforms high-dimensional, complex PDEs into algebraic systems, reducing computational cost while maintaining high accuracy and reliable convergence compared to traditional numerical methods. High-dimensional PDEs often don't have analytical solutions, and traditional numerical methods struggle with the "curse of dimensionality." Wavelet approaches like Bernoulli wavelets can efficiently represent multi-dimensional behavior and capture localized features of solutions without needing extremely fine meshes. Many important phenomena cannot be accurately described with just one or two dimensions (for example Fluid flow in 3D, heat transfer in complex materials, Option pricing models (e.g., Black–Scholes extensions). The solution of high-dimensional PDEs required because they allow us to accurately model, predict, and optimize complex real-world systems that depend on many interacting variables—enabling safer designs, better decisions, and more efficient technologies. This study has applications in following fields:

- Physics and Engineering
- Finance and Economics
- Biology and Medicine
- Artificial Intelligence and Data Science
- Engineering Design and Optimization

4 Problem Statement

The numerical solution of high-dimensional partial differential equations (PDEs) is computationally demanding using classical discretization techniques. Although wavelet-based methods provide sparse representations and multiresolution analysis, the construction of multidimensional Bernoulli wavelet bases and corresponding operational matrices for spatial and temporal derivatives remains underdeveloped. Furthermore, the stability, convergence, and error properties of such schemes have not been rigorously analyzed. This study aims to develop a stable and efficient Bernoulli wavelet–based numerical framework for high-dimensional PDEs and to validate its performance through theoretical analysis and benchmark numerical experiments.

5 Objectives of Research

- ► To develop a Bernoulli wavelet–based numerical scheme for the efficient solution of high-dimensional partial differential equations.
- ► To construct multidimensional Bernoulli wavelet bases and corresponding operational matrices for temporal and spatial derivatives.

- To analyze the stability, convergence, and error behavior of the proposed numerical scheme.

- To validate and assess the performance of the proposed method through benchmark numerical experiments and comparative studies with existing numerical and wavelet-based techniques.

6 Methodology

The following stages would compose the research approach to be used:

- **Stage 1:** Mathematical Preliminaries and Problem Formulation.

- **Stage 2:** Construction of Bernoulli Wavelet Framework.

- **Stage 3:** Development of Operational Matrices.

- **Stage 4:** Numerical Scheme Formulation

- **Stage 5:** Numerical Experiments and Validation.

- **Stage 6:** Comparative Study.

7 Expected Outcomes of the Research

While pursuing Ph.D. program, we expect the following outcomes from our research:

1. Accurate numerical solutions of PDEs.
2. A unified multi-dimensional Bernoulli-wavelet formulation.
3. Efficient and stable computational algorithm.
4. Demonstrated convergence and reliability.
5. Superior accuracy compared to existing methods.
6. Scalability to higher dimensions.
7. Contribution to wavelet-based numerical methods.

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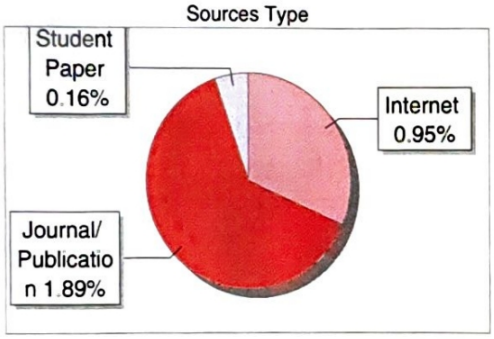

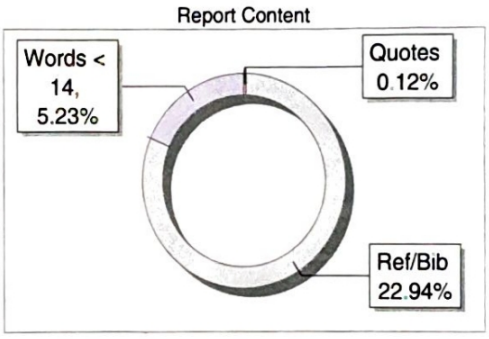

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Numerical solution of high-dimensional partial differential equations using Bernoulli Wavelets

SHORT SYNOPSIS

by

Priyanka

Registration No. : **MRUMADN012K220057**

Under the supervision of

Dr. Aparna Vyas

Associate Professor

ABSTRACT

This work focuses on solving high-dimensional partial differential equations with various initial and boundary conditions using Bernoulli wavelets within a wavelet-based numerical framework, supported by Kronecker product and vectorization techniques. The reliability and computational performance of the proposed method are assessed through CPU time, absolute error estimation, and convergence analysis. The synopsis is divided into seven sections.

Section 1 Presents a concise overview of the historical development of wavelet theory and is organized into four subsections. **Subsection 1.1** Provides the basic definitions of wavelets together with examples, offering important foundational knowledge for the reader. **Subsection 1.2** Presents multi-resolution analysis (MRA), the development of wavelets from MRA. **Subsection 1.3** Provides an overview of wavelet-based numerical methods applied to differential

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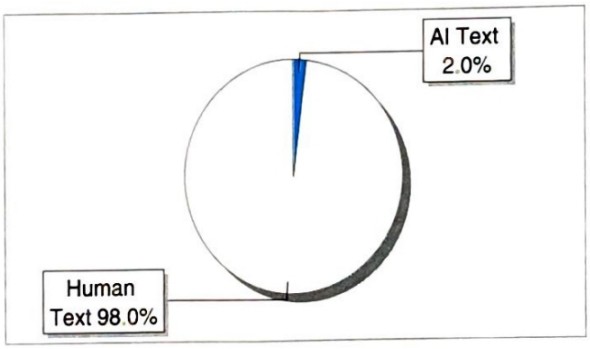

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